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Square Root Cubature Kalman Filter-Kalman Filter Algorithm for Intelligent Vehicle Position Estimate

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Abstract

A new filtering algorithm, adaptive square root cubature Kalman filter-Kalman filter (SRCKF-KF) is proposed to reduce the problems of amount of calculation, complex formula-transform, low accuracy, poor convergence or even divergence. The method uses cubature Kalman filter (CKF) to estimate the nonlinear states of model while its linear states are estimated by the Kalman filter (KF). The simulation and practical experiment results show that, compared to the extended Kalman filter (EKF) and unscented Kalman filter (UKF).The modified filter not only enhances the numerical stability, guarantees positive definiteness of the state covariance, but also increases accuracy, which has high practicability.

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Keywords: square root cubature Kalman filter; Kalman filter; integrated filter; intelligent vehicle

1. Introduction

Research has shown that the fundamental causes of traffic accident occurring lie in the subjective judgment of driver, about 90% of traffic accidents from the driver's mistake (e.g., Fachinger et al. [1]). In order to improve traffic safety and transportation efficiency, intelligent vehicle receives much recognition in many developed countries, which brings it a rapid development. The premise of driving safety is proving precise navigation information for intelligent vehicle (e.g., Nobe et al. [2]). In short the research of vehicle navigation system is very important. Vehicle navigation is often referred as a nonlinear state estimation problem. The extended Kalman filter (EKF) plays a key role in nonlinear state estimation for decades (e.g., Nobe et al. [2]). However, one of the limitations of

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EKF is the calculation of Jacobian matrices [3], which are the first order approximations of nonlinear process and measurement models. Additionally, EKF has truncation errors, it works well only in which the linear approximation of the nonlinear dynamic system and observation model is valid, that narrows the range of EKF application in practical nonlinear systems [4]. In the past years, a few researchers have focused on advanced algorithms for nonlinear state estimation; for example, sigma-point filters such as unscented Kalman filter (UKF) in Julier and Uhlmann, and cubature Kalman filter (CKF) in Arasaratnam and Haykin, etc [2]. The UKF has attracted the most attention in this area, which the deterministic sampling approach is used to capture the mean and covariance [3]. It is shown that a UKF performs much better than EKF, but its run time is considerably longer [5], due to UKF use deterministic sigma-points to capture the mean and covariance, formative time lags [6]. Furthermore, the UKF needs an additional scaling parameter, for negative, when the number of states is more than 3, the UKF may halt its operation [5]. At the same time, parameters should accurate setup, it is inconvenient to operation from a practical point of view. Although CKF uses deterministic sigma-points as well, CKF only requires 2n cubature points [6]. Theoretically, CKF has better computational speed. Therefore, proposed algorithm fusion frame in CKF

During the real-time implementation of KFs, the propagated error covariance matrices may become ill conditioned. These types of ill-conditioned covariance matrix may cause numerical instability during the on-line implementation, even cause the filtering divergence [7]. To handle these difficulties, one can use square-root filters, which propagate and update square root of the error covariance with performing Cholesky decomposition at each step , and they are ensure positive semi-definiteness of the state estimation covariance matrix [8]. Some of the key properties of square-root filters are positive definite error covariance and improved numerical accuracy, etc [9].

2. Square root cubature kalman filter -kalman filter

In this subsection, the derivative square-root cubature Kalman filter-Kalman filter for nonlinear systems is derived. It means that, the proposed algorithm is derived from the square-root CKF and KF, the square-root CKF is used to complete nonlinear state estimation and the KF filter can deal with linear state estimation. The square-root cubature filter-Kalman filter has strong stability and high precision. Consider the following process and observation models [8].

$$\begin{cases} x_k = f(x_{k-1}) + w_{k-1} \\ z_k = h(x_k) + v_{k-1} \end{cases} \tag{1}$$

where x_k and z_k represent the state of the system and measurement at time instant k , respectively. $f(\bullet)$ and $h(\bullet)$ are known vector mappings, $w_k \sim N(0, Q_k)$ and $v_k \sim N(0, R_k)$ are the state and measurement white noises respectively and are mutually independent. The cubature rule to approximate an n-dimensional Gaussian weighted integral is：

$$I(f) = \int_{R^n} f(x)\exp(-x^T x)dx. \tag{2}$$

where $f(x)$ is the arbitrary function. R^n is domain of integration. x^T is the transposed of x .The integral equation is difficult to solve in the general condition. Approximation for a set of points with weight $(\omega_{i,}\xi_i)$ as follow

$$I(f) = \sum_{i=1}^{m} \omega_i f(\xi_i) \tag{3}$$

CKF select 2n cubature points based on spherical-radial criterion, n denote the dimensions of state.

where ξ_i is the i th cubature point located at the intersection of the unit sphere and its axes, ω_i is corresponding weight to cubature point. Thus it can be seen that CKF use $2n$ points which are equally likely to calculated by Gaussian integration directly.

Then, the square root cubature Kalman filter-Kalman filter for state estimation problem contains the following sections.

$$\begin{cases} \xi_i = \sqrt{\dfrac{2n}{2}}[1]_i \\ \omega_i = \dfrac{1}{2n} \\ i = 1, 2, ..., 2n \end{cases} \tag{4}$$

2.1. Initialization

Initialization the filter with initiate state and square root of covariance matrix

$$\hat{x}_0 = E[x_0] \tag{5}$$

$$S_0 = chol\{[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]\} \tag{6}$$

where chol{ · } stand for Cholesky factorization

Generate cubature point ξ_i

Generate weights ω_i

2.2. Time update

Evaluate the cubature points

$$X_{i,k-1} = S_{k-1}\xi_i + \hat{x}_{k-1}, i = 1, 2, \ldots, 2n \tag{7}$$

Propagated cubature points

$$X^*_{i,k} = f(x_{i,k-1}) \tag{8}$$

Predicted state

$$\hat{x}_k = \phi_{k,k-1}\hat{x}_{k-1} \tag{9}$$

Squared-root factor of the predicted error

$$S_k^- = tria([\lambda_k^* \ S_{Q_{k-1}}]) \tag{10}$$

where $tria(\bullet)$ denotes triangular decomposition, S is a lower triangular matrix. $S_{Q_{k-1}}$ denoted a square-root factor of Q_{k-1} .

The weight

$$\lambda_k^* = \frac{1}{\sqrt{2n}}[X^*_{1,k} - \hat{x}_k^- \quad X^*_{2,k} - \hat{x}_k^- \quad \cdots \quad X^*_{2n,k} - \hat{x}_k^-] \tag{11}$$

2.3. Measurement update

Evaluate the cubature points

$$X_{i,k}^{*} = S_k^{-}\xi_i + \hat{x}_k, i = 1, 2, \dots 2n \tag{12}$$

Propagated cubature points

$$Z_{i,k} = h(X_{i,k}^{-}) \tag{13}$$

Predicted measurement

$$\hat{z}_k = \sum_{i=1}^{2n} \omega_i Z_{i,k} \tag{14}$$

Square-root of the Cholesky factorization

$$S_{zz,k} = tria([\lambda_k \quad S_{R_k}]) \tag{15}$$

Cross-covariance matrix

$$P_{XZ,K|k-1} = \frac{1}{\alpha_k}\gamma_k \lambda_k^{T} \tag{16}$$

where

$$\lambda_k = \frac{1}{\sqrt{2n}}[\mathrm{Z}_{1,k}^{*} - \hat{z}_k \quad \mathrm{Z}_{2,k}^{*} - \hat{z}_k \quad \cdots \quad \mathrm{Z}_{2n,k}^{*} - \hat{z}_k] \tag{17}$$

$$\gamma_k = \frac{1}{\sqrt{2n}}[X_{1,k}^{-} - \hat{x}_k^{-} \quad X_{2,k}^{-} - \hat{x}_k^{-} \quad \cdots \quad X_{2n,k}^{-} - \hat{x}_k^{-}] \tag{18}$$

$$\alpha_k = \begin{cases} \lambda_{0,k}, \lambda_{0,k} < 1 \\ 1, \lambda_{0,k} \geq 1 \end{cases}, 0 < \alpha_k \leq 1 \tag{19}$$

$$\lambda_{0,k} = \frac{tr\left(\tilde{y}_k \tilde{y}_k^{T}\right)}{tr\left(\sum_{i=1}^{2n} \omega_i^{(c)} \left(\chi_{i,(k,k-1)} - \hat{Z}_{k,k-1}\right)\left(\chi_{i,(k,k-1)} - \hat{Z}_{k,k-1}\right)^{T}\right)} \tag{20}$$

α_k is the adaptive fading factor which can balance the weight of the equation of state, predictive information and measuring information. Effectively, it would reduce the impact of disturbance.

Filter gain of SRCKF-KF

$$W_k = (P_{xz,k|k-1} / S_{zz,k}^{T}) / S_{zz,k} \tag{21}$$

Updated state based on the new measurement

$$\hat{x}_k = x_k^{-} + W_k(z_k - \hat{z}_k) \tag{22}$$

Square-root factor of the error covariance

$$S_k = tria([\gamma_k - W_k\lambda_k \quad W_k S_{R_k}]) \tag{23}$$

3. Simulations

In order to show the effectiveness of the proposed algorithm, it is an important requirement that all filters have the same initialization and two scenarios are included. The first scenario deals with the state estimation of a class of nonlinear system and the second one deals with the high order nonlinear integrated navigation system.

To quantitatively evaluate the filtering performance, we define the root-mean-square-error (RMSE) in position as

$$RMSE = \sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}\left\|\hat{X}_k^a - X_k^a\right\|^2\right)} \tag{24}$$

where X_k^a and $\hat{X}_k^a$ are the actual and the estimated positions at the Monte Carlo run, respectively. Similar to the RMSE in position, we can easily write the RMSE in velocity and acceleration. The smaller value of RMSE means the higher precision of the filter algorithm.

3.1. A class of nonlinear system

Consider the nonlinear non-Gaussian system with one-dimensional state which can be modeled by the Equations (25) [7]

$$\begin{aligned} x_{k+1} &= 0.5x_k + \frac{25x_k}{1+x_k^2} + 8\cos[1.2(i-1)] + w_k \\ z_k &= x_k^2 + v_k \end{aligned} \tag{25}$$

where w_k and v_k are the white noise with zero mean and a nonsingular covariance. The variance values of w_k and v_k is 10 and 0.001 Fig. 1 shows the true value (green line) and the estimate of EKF (red line), UKF (blue line) and SRCKF-KF (black line).

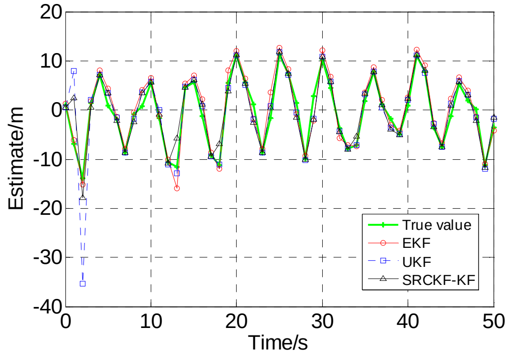

Fig. 1. true value and estimate.

As mentioned previously, the SRCKF-KF (black line) is superior to the EKF (red line) and UKF (blue line), cause the limited of coordinate, the differences were not significant from the Fig. 1, just in the first ten seconds the blue line (UKF) deviates from the green line (true value) distinctly, and around 13 seconds, both red line (EKF) and blue line (UKF) made bigger deviation than other time. Overall, black line (SRCKF-KF) is the nearest line to the green line (true value).

In order to quantitative analysis the difference of different algorithms, we make 100 independent Monte Carlo (MC) runs and calculate he RMSE in position, which is shown in Fig. 2 and after a number of 100 MC runs, the RMSE and computational costs of a time step (100 time step in all) are given in Tab.1

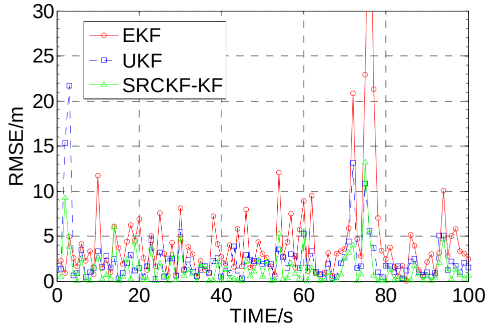

Fig. 2. RMSE of several methods in Example 1.

Table 1. Performance comparison in Example 1.

Method	RMSE	Time(s)
EKF	2.8501	0.0057
UKF	2.2405	0.0253
SRCKF-KF	1.9350	0.0152

In Fig. 1 estimate value of SRCKF-KF is the closest with the true value. In Fig. 2 the RMSE using the SRCKF-KF filter is smaller than the EKF or UKF. Quantitative estimation precision and computational complexity of the algorithm are given in Table 1. Obviously, the result indicated that the SRCKF-KF has the better performances on the estimation accuracy, compared with UKF, the relative improvement of SRCKF-KF average precision is nearly 10.8%. But compared with the filters response, EKF has better performances on the computation simplicity. Both UKF and SRCKF-KF are computationally expensive, because they are deterministic sampling filter, the time cost is higher, but SRCKF-KF need less sampling points than UKF. Although the difference of simulation is not obvious, as the sampling point increase, UKF would be at least one order of magnitude slower than SRCKF-KF and this difference will have significant negative impact on actual engineering.

3.2. Integrated navigation system

In this subsection, to ensure a fair and reasonable performance comparison of the different filtering algorithms, the state estimation of GPS/INS will be considered. Vehicle is approximated here as a 2-D point mass, moving with the limited speed and turning rate. And it was located by relative location. In order to transform conveniently, UTM (Universal Transverse Mercator Projection) coordinates are adopted in this system to display the information the position of vehicle, the systematic equation is

$$x_{k+1} = F_{k+1} x_k + u_k + w_k \tag{26}$$

where, u_k is the known input control vectors, $u_k = I$ in simulation and I stands for unit matrix; w_k is white Gaussian noise, and the state variables are proposed as follow [10]:

$$X_k = [s_e, s_n, v_e, v_n, a_e, a_n]^T \tag{27}$$

Where, each component are due to the position, the velocity and acceleration of two directions of east and north, respectively. The units are: m, m, m/s, m/s, m/s^2, m /s^2;

The transition matrix is

$$F_{k+1}=\begin{bmatrix}1 & 0 & T & 0 & T^2/2 & 0\\ 0 & 1 & 0 & T & 0 & T^2/2\\ 0 & 0 & 1 & 0 & T & 0\\ 0 & 0 & 0 & 1 & 0 & T\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix} \tag{28}$$

The measurement equation is

$$z_{k+1}^{gps}=h_{k+1}^{gps}(x_{k+1},u_{k+1})+v_{k+1}^{gps} \tag{29}$$

where, $h_{k+1}^{gps}=[h_{k+1,e}^{gps},0,0,h_{k+1,n}^{gps}]$ is stand for the measurement matrix of GPS, $h_{k+1,n}^{gps}=[1,0,0;0,1,0]$.

The model is initialized at $X_0=[237 \quad 68 \quad 0 \quad 0 \quad 0 \quad 0]^T$ and the process and measurement noises are

$$Q_k=T_{a1}*T_{a1}^T*i^2 \tag{30}$$

$$T_{a1}=\begin{bmatrix}T^2/2 & 0 & T & 0 & 1 & 0\\ 0 & T^2/2 & 0 & T & 0 & 1\end{bmatrix},T=0.1 \tag{31}$$

where T is sample time, v_{k+1}^{gps} is assumed as white Gaussian distributed with zero mean. The corresponding results of the different filter algorithm's estimate are shown in Fig. 3, which red dotted line stand for the estimated value of EKF, similarly, green dotted line stand for the estimated value of UKF, black dotted line stand for SPCKF-KF and blue line means the real trajectory. It should be clear that black dotted line (SPCKF-KF) get the closest points of approach. In other words, simulation results prove that the proposed algorithm exhibits excellent performances, particularly in terms of turn lane tracking. While the green dotted line (UKF) in simulation, it does reveal time-delay problems of UKF obvious. In addition, we use RMSE to visualize the errors of each filter algorithm. The RMSEs of independent component of east using the EKF, UKF and SRCKF-KF filter are shown in Figs 4 and 5. Due to the limitation of length, independent component of north is not set out. The result of simulation shows that the SRCKF-KF had higher accuracy and quicker convergence speed than existing method.

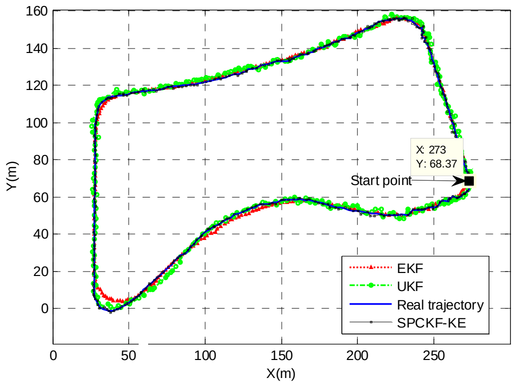

Fig. 3. real trajectory and estimate.

Fig. 4(a) shows that the clear distinctions of estimate error in position between three kinds of algorithms, the RMSE of EKF even more than 10 m, while the RMSE of SRCKF-KF less than 5 m totally and even more precise. In the aspect of velocity estimation which shows in Fig. 4(b), there is no great difference between three algorithms, however we still can separate SRCKF-KF from others, because the estimate of SRCKF-KF generates only one point big error value more than 0.25 m/s, EKF has two error values more than 0.25 m/s and UKF has more. Fig. 4(c) shows the advantages of the proposed algorithm, both EKF and UKF has produce rather large error in the first 100 seconds, on account of the vehicle accelerate from a standstill, this kind of situation is also be the case in the last 50 seconds, cause vehicles have to lower the speed and stop, nevertheless, SRCKF-KF shows excellent performances in this two periods of time. Hence, the proposed algorithm filter shows the superiority in all respects, which would appear to be well suited for non-linear state estimation.

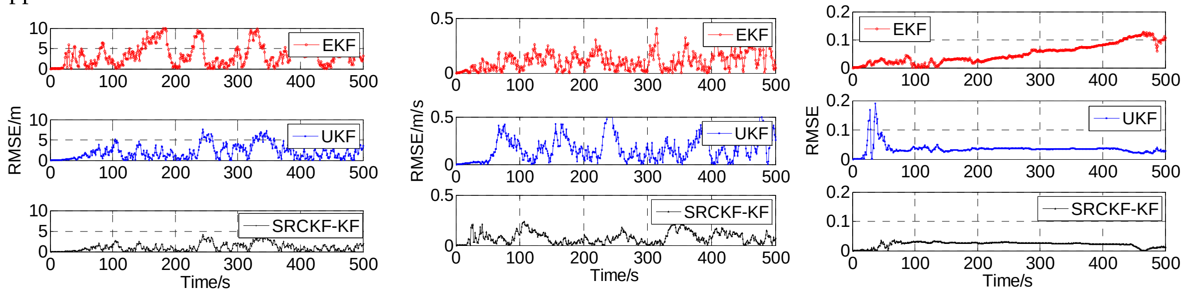

Fig. 4. (a) RMSE in position; (b) RMSE in velocity; (c) RMSE in acceleration.

4. Experiments

In order to validate the application performance of proposed algorithms, this section presents a validation of the method, which is carried out by substantial vehicle tests.

Take the Beijing University of Technology intelligent vehicle (BJUT-IV) as the experiment platform to verify the new algorithm’s feasibility and superiority. Fig. 5 shows the platform of intelligent vehicle (BJUT-IV equipped with several sensors which have been marked in the figure). “BJUT-IV” is an electrical customized car, which was refitted from electric golf carts with improvements. BJUT-IV moves at a constant velocity or constant acceleration, we approximate the uniform motion of turning as the uniform motion without influence the testing accuracy.

Fig. 5. Beijing University of Technology-intelligent vehicle (BJUT-IV).

The azimuth information of vehicle acquired by GPS/INS is used to estimate the distance, speed and other motion information of the vehicle through non-linear filtering. BJUT-IV equipped with GPS carrier-phase measuring technique. These equipments were composed of GPS receiver of FlexPak-G2 series, antenna of GPS-702-GG series and data transmission broadcasting station of PDL4535 series. Fig. 5 shows the equipments were used in the BJUT-IV.

BJUT-IV adopts integrated navigation (GPS/INS) as the navigation system. For accurately navigation, UTM coordinates was used to transform Geographic Coordinate system into 2D rectangular coordinate system. Reset N39.8719°, E116.4789° as origin of coordinates, marked as N8.

We designed a software platform used for the test and verification of performance of different algorithm for filtering. The experimental platform was created by the combination of the VC++6.0, MapX, Matlab and Matcom. The platform realizes fusion and visualization of multi-sensor data.

To process experimental date, KF (green line), EKF (yellow line) and UKF (blue line), SRCKF-KF (red line) filters were used respectively. Orange line stands for the actual vehicle driving road. On the condition of selecting East-North-Up coordinate as navigation coordinate, position, velocity and acceleration of BJUT-IV are taken as state variable matrix under rectangular coordinate system, which is described as follow

$$X = [x \quad y \quad \dot{x} \quad \dot{y} \quad \ddot{x} \quad \ddot{y}]^T \tag{32}$$

where, matrix elements are the position, velocity and acceleration components of east and north, respectively. Fig. 6 shows the estimated position from different filters, and the enlarged drawing of the corners.

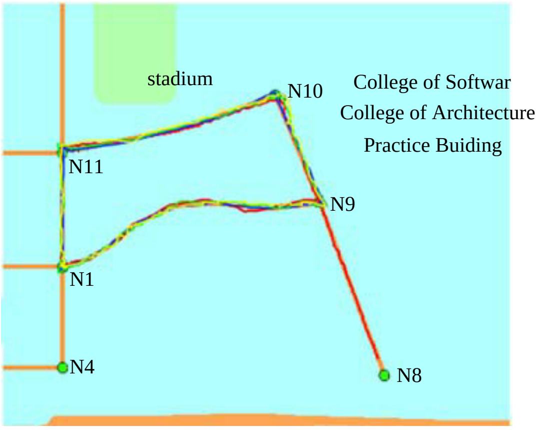

Fig. 6. actual and estimated states.

Obviously, the screenshots of experiment shows that in the vehicle position estimation, the better results were achieved by proposed algorithm filter (red line), the position estimated by SRCKF-KF has flatter response curve, especially for the turn in the road. As opposed to proposed algorithm, KF's (green line), EKF's (yellow line) and UKF's (blue line) performance were less than SRCKF-KF. On the one hand, both EKF'S and KF's (green line) estimated trajectory is not stable, on account of EKF is just simple nonlinear filtering, while KF is just linear filtering, neither EKF nor KF can adapt to the integrated navigation system, notably at the turning marked as N10, on the other hand, the blue line (UKF's estimated value) presents an obvious hysteretic nature , due to the sampling points of UKF are more than those of SRCKF-KF, this experimental result agrees with the theoretical analysis. Based on above analysis, the proposed algorithm was proved to be reliable and accurate through testing cars in practice.

5. Conclusions

In this paper, a square-root cubature filter with Kalman filter is developed for nonlinear systems. The proposed filter is derived from an extended square-root cubature filter fused with a Kalman filter, which uses cubature Kalman filter (CKF) to estimate the nonlinear states of model while its linear states are estimated by the Kalman filter. The advantages of the proposed algorithm are as follows:

- It does not require the evaluation of Jacobians for nonlinear state estimation, which has low computing complexity . In many time-sensitive engineering applications, it can give quick but good approximations of much more complicated calculations.
- In the process of filtering by means of square root of covariance matrix directly. It is guarantees positive definiteness of the state covariance while keeping high accuracy and avoiding the divergence
- It uses CKF estimate nonlinear state and KF estimate linear state, which is more logical and flexible for intelligent vehicle integrated navigation system.

- It has excellent performance in practical testing. Owing to adaptive fading factor, SRCKF-KF is effective with its rapid response speed and strong robustness which has certain significance for the solution of practical engineering problems.

Next research steps include introduction of fading factor to improve the adaptability, then reinforcement algorithm with noise statistic estimator to solve the problem that the algorithm declines in accuracy and further diverges when the prior noise statistic is unknown and time-varying.

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